The Acoustic Bubble

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$$\{\mathrm{IL}\} = 10\log_{10}\left(\frac{I}{I_{\mathrm{ref}}}\right) \tag{1.40}$$

The ratio is that of the sound intensity I to some reference intensity I_{ref}. Since the intensity is proportional to the square of the pressure amplitude, this measure is equivalent to $20\log_{10}(P_1/P_2)$ where P_1 and P_2 are the acoustic pressures of the two signals. For convention, the *sound pressure level* {SPL}, in decibels, is taken as the ratio of the acoustic pressure to a reference P_{ref}

$$\{\mathrm{SPL}\} = 20\log_{10}\left(\frac{P}{P_{\mathrm{ref}}}\right) \tag{1.41}$$

Obviously the acoustic pressure and the reference must be measured in the same way (e.g. amplitude, or r.m.s.[22] pressure). If the reference intensity and the reference pressure represent the same physical wave, then IL is equivalent to SPL.

In air, the reference standard is taken as $I_{\mathrm{ref}} = 10^{-12}$ W m^{-2}, which is approximately the threshold intensity for normal human hearing at 1 kHz. This threshold corresponds to an acoustic pressure amplitude of 28.9 μPa (equation (1.39)) for plane and spherical travelling waves. The SPL is usually taken from the ratio of the r.m.s. acoustic pressure to a reference pressure of 20 μPa (which is the nearest integer μPa corresponding to the I_{ref} intensity, the r.m.s. acoustic pressure of a sinusoidal wave having amplitude $P_A = 28.9$ μPa being 20.4 μPa). Because of this rounding, SPL is almost, but not exactly, equal to the IL for plane and spherical waves. In underwater acoustics, reference pressures of 20 μPa, 1 μbar and 1 μPa, equivalent to intensities of 2.70×10^{-16}, 6.76×10^{-9} and 6.76×10^{-19} W m^{-2} respectively, are used. The latter is now the more common [16]. However, use of SPL and IL should be accompanied by the quoted reference pressure.

If the waveform is more complicated than the simple plane and spherical travelling waves, for example, in standing wave fields where equation (1.39) does not hold, the measurements of SPL and IL can disagree. In most situations when the interaction of such a sound field with a bubble is considered, the size of the bubble is significantly less than the lengthscale over which significant pressure changes occur, and the response time of the bubble is less than or comparable with the acoustic period. The behaviour of an individual bubble is therefore determined by the instantaneous value of the local pressure field, and it is more appropriate to refer to the acoustic pressure measurement; hydrophones[23] give an instantaneous voltage representation of the local field.

The advantage of this scale is that it is logarithmic, and so can more readily express the vast range in intensities (approximately fourteen orders) to which our hearing can respond. In addition, the human sensory perception of loudness is logarithmic, in that we judge one sound to be so many *times* louder than another [17].

1.1.4 Radiation Pressure

In the preceding section the energy associated with a wave was calculated. The wave transmits energy, and the absorption of that energy will generate a force upon the absorber.

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[22] r.m.s. means 'root mean square', which is calculated by squaring the acoustic pressure over some interval, finding the mean of this, then taking the square root of that mean so the result has dimensions of pressure. For a sinusoidal wave, the r.m.s. pressure is $1/\sqrt{2}$ times the acoustic pressure amplitude.

[23] See section 1.2.2(a)(i).

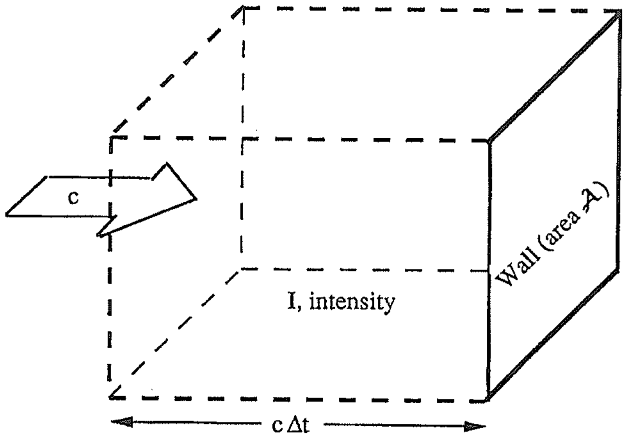

Figure 1.7 The energy in a plane travelling wave approaches a plane of area A, which is perpendicular to the direction of motion of the wave, at speed c.

Consider again the plane wave, travelling in the $+x$ direction, approaching a wall in the yz-plane of area A (Figure 1.7). The wave energy is completely absorbed by the wall. If the wave has intensity I, then the energy absorbed by the wall in a time Δt is $IA\Delta t$. The wall must have applied a force F_r in the $-x$ direction to stop the wave motion, which in time Δt acted over a distance $c\Delta t$. Therefore the work done by the wall on the wave is $F_r c\Delta t$. Equating this to the energy absorbed, we obtain $F_r = (IA/c)$. From Newton's Third Law of Motion, this must be equal and opposite to the force exerted by the wave on the wall. Therefore upon absorption the wave exerts a *radiation pressure* in the direction of its motion, of magnitude

$$p_{\text{rad,abs}} = \frac{I}{c}, \qquad \text{for normal incidence of plane waves.} \qquad (1.42)$$

The force F_r exerted by the wall can also be thought of as acting upon the wave to absorb its momentum. In time Δt the wall absorbs a length $L = c\Delta t$ of the wave, exerting an impulse $F_r\Delta t = IAL/c^2$ upon the wave, causing a change in momentum of Δp. Since after absorption the momentum of wave is zero, then the momentum associated with one wavelength of the wave is

$$p_\lambda = \frac{IA\lambda}{c^2} \qquad (1.43)$$

If the wave is reflected, instead of being absorbed, this momentum must be not simply absorbed but reversed. The wall must exert twice as much force upon the wave, and so the radiation pressure felt by the reflector is

$$p_{\text{rad,refl}} = \frac{2I}{c} \qquad (1.44)$$

for total reflection of normally incident waves back along the line of incidence.

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Beissner [18] calculates
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Figure 1.8 A travelling wave
original direction of motion o

[24]A baffled source is one very c
1.2.1(a), and Chapter 3, sections

If a wave is partially reflected and partially absorbed, the radiation pressure is intermediate between the two values given by equations (1.42) and (1.44). In one practical case, where the radiation pressure is used to measure the acoustic intensity (see section 1.2.2(a)(ii)), the sound may be incident at 45° to the reflector. The wall exerts the force F_r in the direction of incidence (to absorb the wave momentum in that direction), and exerts another force of equal magnitude perpendicular to the incidence, to generate the momentum for the reflected wave (Figure 1.8). The total radiation force is therefore $\sqrt{2}F_r$ normal to the reflector, giving an appropriate radiation pressure of $\sqrt{2}(I/c)$.

Beissner [18] calculates the radiation pressure resulting from geometries other than the plane wave, where diffraction effects must be considered. The simplest of these results is for a baffled[24] circular plane piston of radius L_s, which gives

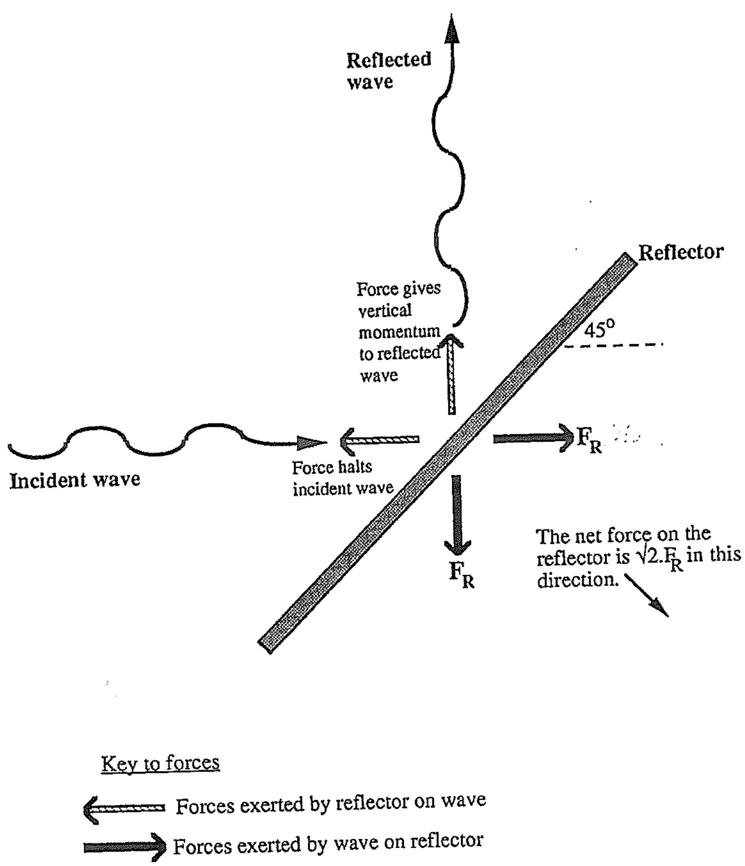

Figure 1.8 A travelling wave is reflected through 90° by a perfect reflector which is angled at 45° to the original direction of motion of the wave.

[24] A baffled source is one very close to an infinite rigid boundary, which thus emits into a half-space (see section 1.2.1(a), and Chapter 3, sections 3.3.2(a) and 3.3.2(b)).

$$p_{\mathrm{rad,abs}} = \frac{I}{c}\left\{\frac{1 - J_0^2(kL_s) - J_1^2(kL_s)}{1 - J_1(2kL_s)/kL_s}\right\} \tag{1.45}$$

where J_n is the Bessel function of order n [19].

1.1.5 Reflection

(a) Reflection at Normal Incidence

As with any waveform, sound waves can be reflected at interfaces between two differing media, giving rise at audio frequencies to the familiar echo effect. In acoustics, the criterion which distinguishes the difference between media is the acoustic impedance, as this section will show.

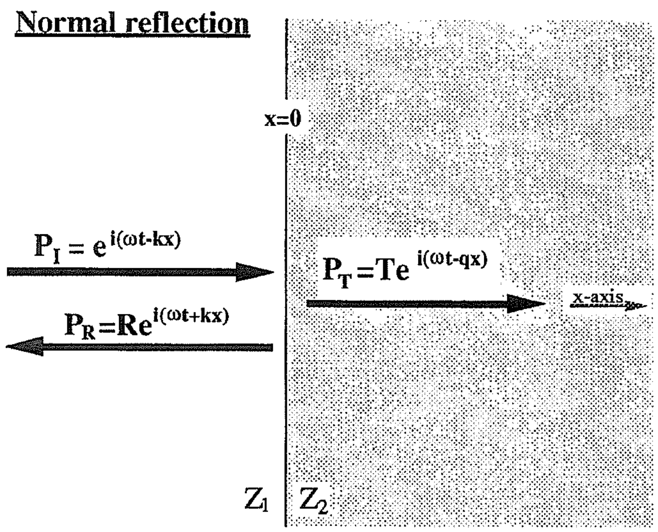

Figure 1.9 A travelling pressure wave, incident normally on a plane boundary, is part reflected and part transmitted.

Consider the interface shown in Figure 1.9. An incoming sound wave of normalised pressure amplitude $P_I = e^{i(\omega t-kx)}$ propagates in medium 1 parallel to the x-axis and is reflected at the boundary (at $x = 0$) with medium 2. A reflected pressure wave $P_R = Re^{i(\omega t+kx)}$ travels back into medium 1 and a wave $P_T = Te^{i(\omega t-qx)}$ is transmitted into medium 2. All waves here travel along the x-axis, the reflected wave following the $-x$ direction, and the other two waves the $+x$ direction. R is the pressure amplitude reflection coefficient, and equals the ratio of the amplitude of the reflected to the incident pressure wave. T is the corresponding transmission coefficient, and is numerically equal to the ratio of the amplitude of the transmitted to the incident pressure wave. The specific acoustic impedances of the two media are Z_1 and Z_2. Since there can be no discontinuity in pressure at the massless interface ($x = 0$), then

$$P_I + P_R = P_T \qquad \Rightarrow$$

$$1 + R = T$$

The velocities must match at
from equation (1.27)

$$\left(\frac{1}{Z_1}\right)(1 - R) = \left(\frac{1}{Z_2}\right)T$$

the negative sign before the
travelling in the $-x$ direction.

$$T = \frac{2Z_2}{(Z_1 + Z_2)}$$

and

$$R = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)}$$

for the pressure amplitude tr
If $Z_1 = Z_2$, then the wave
reflected component, and the
an air–water interface. The in
is 1.5×10^6 kg m^{-2} s^{-1}. Ther
amplitude reflection coeffici
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might perhaps be why the co
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160 kHz respectively [1]. Th
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boundary. When $Z_1 \ll Z_2$, then
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equal to T^2).
As in all acoustics, one mu
coefficients are being discuss
ment, it can readily be show
times that for pressure. To ill
cients will now be derived fo

(b) Oblique Reflection

The incident (normalised),
$\varepsilon_I = e^{i(\omega t - kx\cos\theta_I + ky\sin\theta_I)}$, $\varepsilon_R =$